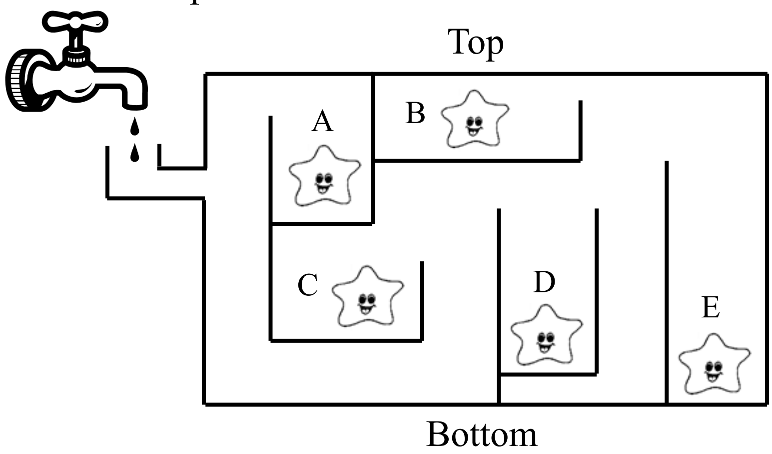
2013 Middle Primary Division Second Round Solution

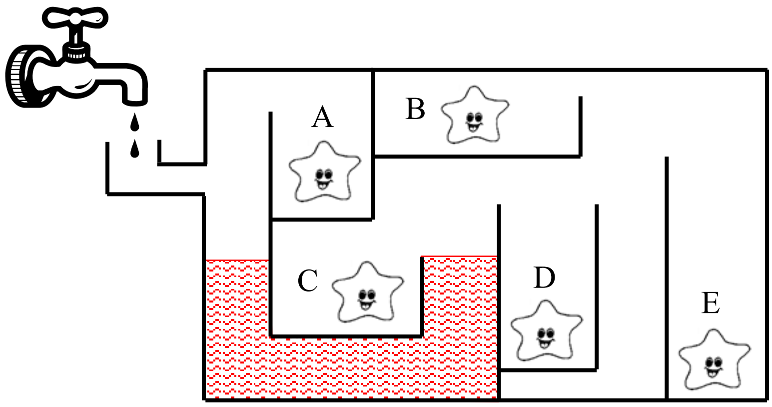
1. The diagram shows an aquarium containing five starfish, each occupying a labelled compartment. Water is pumped into the aquarium through the pipe on the left side. Which compartment is the first to be flooded with water?



（A） A （B） B （C） C （D） D （E） E

【Solution】

The diagram shows the moment when the first compartment is about to be flooded with water. The answer is (C).



Answer：(C)

2. When 10101 is subtracted from 10000000, how many times does the digit 9 appear in the difference?

（A） 3 （B） 4 （C） 5 （D） 6 （E） 7

【Solution】

In the difference $10000000-10101=9989899$, the digit 9 appears 5 times. The answer is (C).

Answer：(C)

3. What is the sum of 32 copies of 1000, 19 copies of 100 and 29 copies of 10?

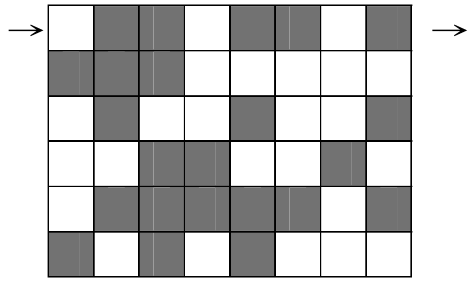
（A） 3219290 （B） 321929 （C） 342190 （D） 34190 （E） 32129

【Solution】

We have $32\times1000+19\times100+29\times10=32000+1900+290=34190$. The answer is (D).

Answer：(D)

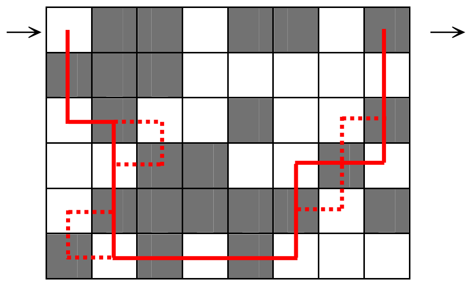
4. The diagram shows a 6 × 8 chessboard with squares painted in black and white in an unusual pattern. Starting from the top left corner, a marker must move between squares which have opposite colours and share a common border. What is the minimum number of black squares it must visit in order to arrive at the top right corner, counting it as one of the black squares visited?



（A）3　　（B）8　　（C）9　　（D）10　　（E）11

【Solution】

The diagram shows a main path which takes the marker over 9 black squares. There are two detours near the bottom left corner, but each involves increasing the number of black squares visited, and are ignored. There are also two places with alternative routes near the bottom right corner, and in each case, either can be taken without affecting the number of black squares visited. The answer is (C).



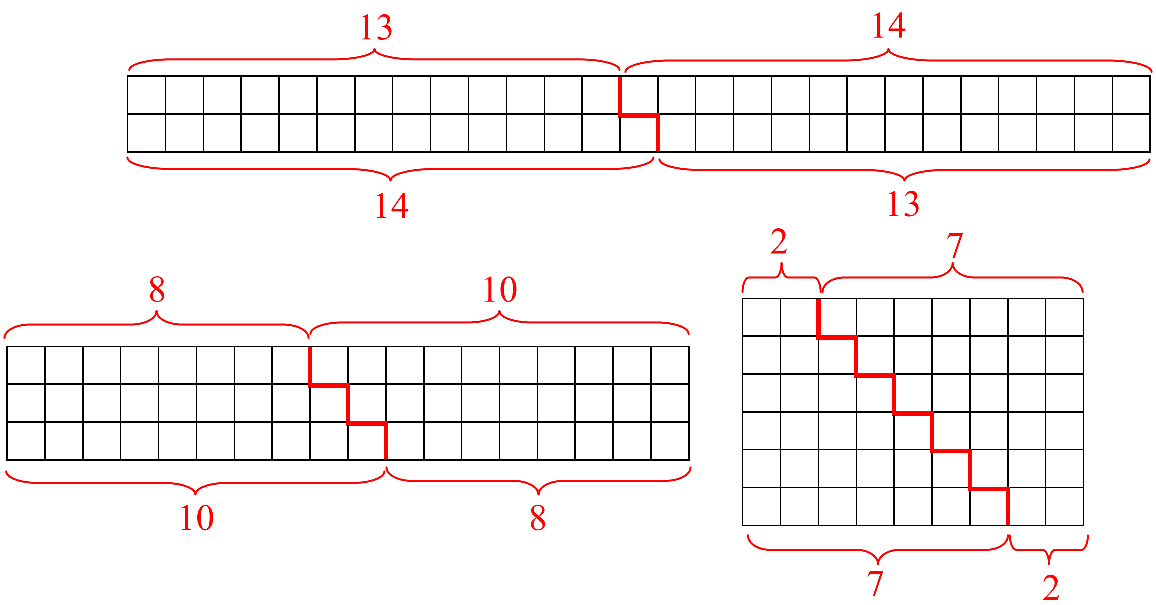
Answer：(C)

5. Max gives 27 apples to a group of friends. The numbers of apples they receive are consecutive positive integers. What is the maximum size of this group?

（A）2　　（B）3　　（C）4　　（D）5　　（E）6

【Solution 1】

A sum of consecutive positive integers may be represented geometrically as a staircase. Two identical staircases can be put together to form a rectangle. If the sum of the consecutive integers is 27, the area of the rectangle is 54. Not counting the 1 × 54 rectangle, there are three others with integral dimensions, namely, 2 × 27, 3 × 18 and 6 × 9. They are shown in the diagram, partitioned into two identical staircases. The sums they generated are 13+14, 8+9+10 and 2+3+4+5+6+7. The answer is (E).



【Solution 2】

There are three ways of expressing 27 as a sum of consecutive positive integers, namely, 27 = 13+14, 27 = 9+9+9 = 8+9+10 and 27 = 9+9+9 = (4+5)+(3+6)+(2+7) = 2+3+4+5+6+7. It follows that 27 can be expressed as a sum of at most 6 consecutive positive integers. The answer is (E).

Answer：(E)

6. When the digits 0, 1, 2, 5, 6, 8 and 9 are rotated 180°, they become 0, 1, 2, 5, 9, 8 and 6 respectively. What does 9105 become when the four-digit number is rotated 180°?

【Solution】

The last digit 5 of the given number becomes the first digit 5 of the number we seek. The other three digits, namely, 0, 1 and 9, become 0, 1 and 6 respectively. Hence the number we seek is 5016.

Answer：5016

7. An ant by itself is unable to drag a slice of bread back to the anthill. So summons 9 other ants to help, but the slice is still too heavy. So each of these 10 ants summons 9 other ants to help, and they manage to drag the slice back to the anthill. How many ants are involved?

【Solution】

The total number of ants involved is $(1+9)+10\times 9=100$.

Answer：100 ants

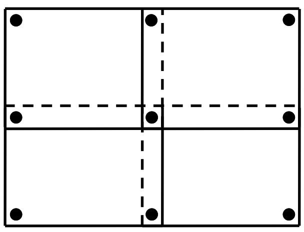
8. Lily has 100 chocolates. She eats one on the first day. Each day after, she eats twice as many as the day before, until all the chocolates have been eaten. How many chocolates did she eat on the last day?

【Solution】

During the first six days, Lily eats respectively 1, 2, 4, 8, 16 and 32 chocolates, Since 1+2+4+8+16+32=64-1=63, 100-63=37 chocolates are left, which is less than 64. Hence the number of chocolates Lily eats on the last day is 37.

Answer：37

9. A class is putting up 10 rectangular posters of the same shape and size on a wall. Each poster must be held in place by one nail near each corner. Adjacent posters may overlap slightly so that the same nail can serve to hold both of them. The diagram shows how 9 nails can hold four posters adjacent diagonally. What is the minimum number of nails required to hold all 10 posters?



【Solution 1】

Clearly, the 10 posters should be arranged in a rectangular array, and there are four ways to do so.

For a 1×10 array, the number of nails required is $(1+1)\times(10+1)=22$.

For a 2×5 array, the number of nails required is $(2+1)\times(5+1)=18$.

Hence the minimum number of nails required is 18.

【Solution 2】

Clearly, the 10 posters should be arranged in a rectangular array, and there are four ways to do so. We wish to maximize the number of four-way common corners.

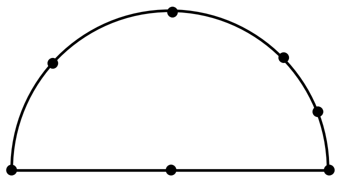
For a 1×10 array, there are $(1-1)\times(10-1)=0$ such corners.

For a 2×5 array, there are $(2-1)\times(5-1)=4$ such corners.

Hence the minimum number of nails required is $4+2\times(2+5)=18$.

Answer：18 nails

10. The diagram shows seven marked points, six on a semicircular arc, including both endpoints of the diameter, along with the centre of the arc. How many triangles are there whose vertices are all chosen from these points?



【Solution1】

If we do not take any of the three points on the diameter as vertices, there are 4 such triangles.

If we take only one point on the diameter as a vertex, this can be chosen in 3 ways. The other two vertices can be chosen in 6 ways. The number of triangles in this case is 3 × 6 = 18.

If we take two points on the diameter as vertices, they can be chosen in 3 ways. The other vertex can be chosen in 4 ways. The number of triangles in this case is 3×4=12.

The total number is 4+18+12=34.

【Solution2】

The first vertex can be chosen in 7 ways, the second one in 6 ways and the third 5 ways, for a total of 210. However, the same triangle can arise from choosing the vertices in different orders. Each triangle arises 3 × 2 × 1 = 6 times, so that the total number is reduced to 210 ÷ 6 = 35. From this, we must still subtract the one in which all three chosen vertices are on the diameter. Hence the final count is 35 − 1=34.

Answer：34 triangles

11. The diagram shows an addition of a three-digit number, a two-digit number and a one-digit number, with a three-digit sum. The same letter stands for the same digit and different letters stand for different digits. A question mark can stand for any digit, including those represented by a letter. What is the maximum value of the sum?

$$\begin{array}{cccc} & X & Y & Z \\ & & Y & Z \\ + & & & Z \\ \hline & ? & ? & ? \end{array}$$

【Solution】

Since the sum is a three-digit number, it is at most 999. If it is 999, then $Z=3$, but we cannot have $2Y=9$. The next largest possible sum is 998. Here we have $Z=6$, $Y=4$, $X=9$, with 946+46+6=998.

Answer：998

12. Leon uses a code to convert a letter string consisting only of As, Bs and Cs, into a number string consisting only of 0s and 1s, by replacing A with 101, B with 11 and C with 0. If the number string obtained is 110101101110101, what is the number of letters in the original letter string?

【Solution】

If the first number of a string is 0, it must stand for C. If it is a 1, and the next letter is another 1, the two 1s must stand for B, If the next letter is 0, then the third letter must be 1 and these three letters must stand for A. Thus the number string can be read in a unique way. In particular, 11-0-101-101-11-0-101 must stand for BCAABCA , and the number of letters is 7.

Answer：7 letters

13. The total number of players on three badminton teams is 29. No two players on the same team play against each other, while every two players on different teams play each other exactly once. What is the maximum number of games played?

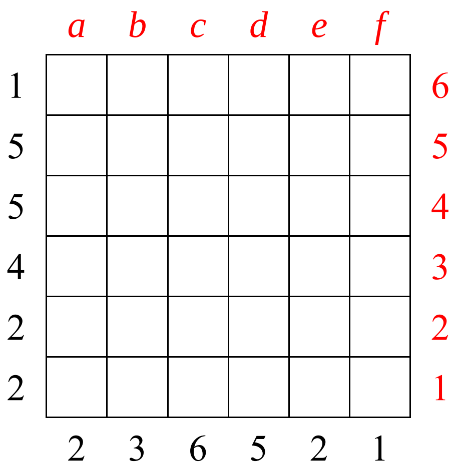
【Solution】

Suppose team A has at least two more players than team B. Transfer one player X from team A to team B. Before the transfer, X plays every player on team B. After the transfer, X plays every player left on team A, which is still more than the number of players originally in team B. So the total number of games played has increased. It follows that to maximize the total number of games played, the size of the teams should not differ by more than 1. Hence we should have 10 players in two of the teams and 9 players in the third team.

The total number of games is then $10 \times 10 + 10 \times 9 + 10 \times 9 = 280$.

Answer：280 games

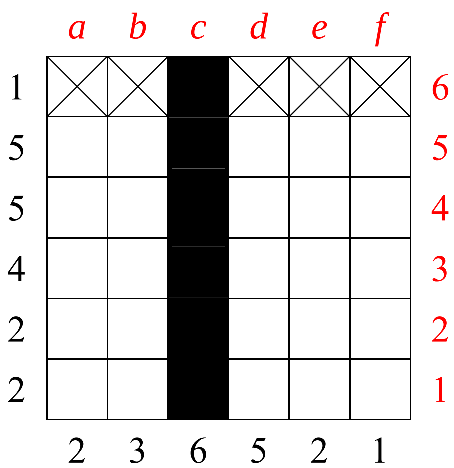
14. Some of the squares in the 6×6 table are shaded. The numbers of shaded squares in the respective rows and columns are indicated on the edge of the table, and there are no gaps between the shaded squares in any row or column. Show where the shaded squares are.



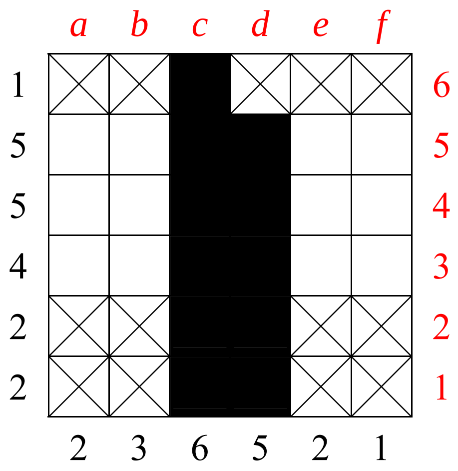
【Solution】

We label the rows 1 to 6 from bottom to top and the columns *a* to *f* from left to right. Obviously, all of *c*1, *c*2, *c*3, *c*4, *c*5 and *c*6 must be shaded, and this means that *a*6, *b*6, *d*6, *e*6 and *f* 6 are not shaded.

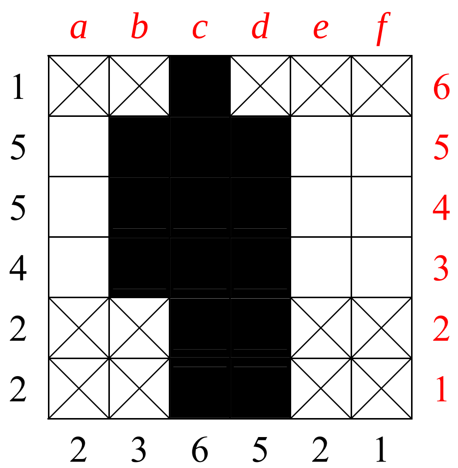
(5 points)



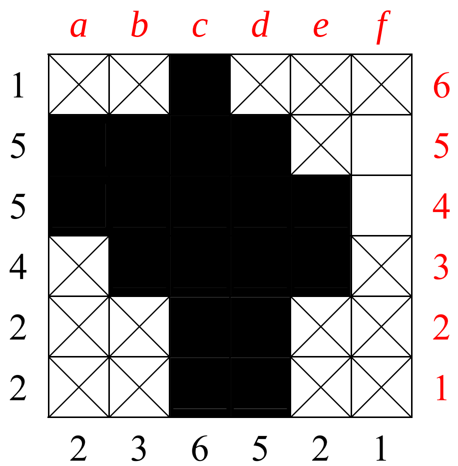
Hence all of $d1$, $d2$, $d3$, $d4$ and $d5$ must be shaded. This means that $a1$, $b1$, $e1$, $f1$, $a2$, $b2$, $e2$ and $f2$ are not shaded. (5 points)



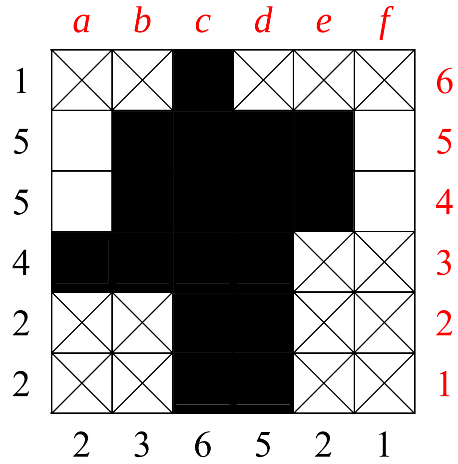
This forces the shading of $b3$, $b4$ and $b5$.



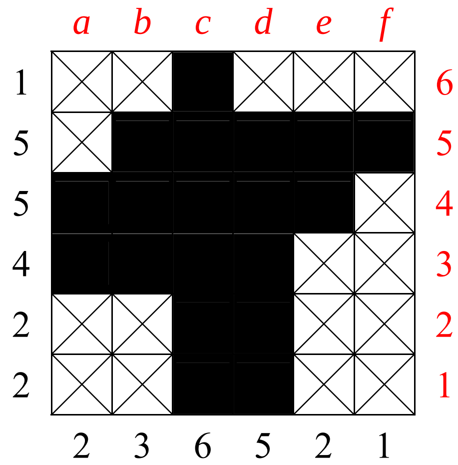
If the last shaded square in row 3 is $f3$, it will be isolated. If it is $e3$, then $a4$ and $a5$ must be shaded. However, the lone square in column f must have a shaded square in the same row in column e, and that row will have too many shaded squares. (5 points)



It follows that *a*3 must be shaded, as well as *e*4 and *e*5.



The last two shaded squares are *a*4 and *f* 5. (5 points)



15. A three-digit number is 13 times the product of its digits. The hundreds digit is larger than either of the other two digits. What is this number?

【Solution】

Since the three-digit number is equal to 13 times the product of its digits, it is divisible by its hundreds digit, which is the largest of the three. If we increase both the tens digit and the units digit to the hundreds digit, the quotient will be 111. If instead we decrease both of them to 0, the quotient will be 100.

The actual quotient is between 100 and 111(5 points), and since it is divisible by 13, it must be 104(5 points). The product of the tens digit and the units digit is $104 \div 13=8$. Since $8=2 \times 4=1\times 8$, the three-digit number is one of 624, 642, 918 and 981(5 points). The only one divisible by 13 is $13\times 6\times 2\times 4= 624$. The number is 624. (5 points)

Answer：624